

微積分 演習 (略解) (情報メディア学科1年次科目)

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3 複素平面とオイラーの公式

3.1 複素数の演算

略解

1. $z_1 = e^1 \times e^{-\frac{\pi}{3}i} = e \cdot (\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3})) = \frac{1}{2}e - \frac{\sqrt{3}}{2}ei$. よって, $\operatorname{Re} z_1 = \frac{e}{2}$, $\operatorname{Im} z_1 = -\frac{\sqrt{3}e}{2}$.

2.

$$|z_2| = \sqrt{(+1)^2 + (-1)^2} = \sqrt{2}. \quad (3.1)$$

$$\operatorname{Arg} z_2 = \frac{7}{4}\pi. \quad (3.2)$$

3.

$$(z_2)^{-10} = \left(\sqrt{2}e^{-\frac{1}{4}\pi i}\right)^{-10} = \frac{1}{2^5}e^{\frac{10}{4}\pi i} = \frac{1}{32} \cdot e^{+2\pi i} \cdot e^{\frac{2}{4}\pi i} = \frac{i}{32}. \quad (3.3)$$

$$\operatorname{Re}((z_2)^{-10}) = 0, \operatorname{Im}((z_2)^{-10}) = \frac{1}{32}.$$

4.

$$z_1 \cdot z_2 = e \cdot e^{-\frac{\pi}{3}i} \times \sqrt{2}e^{-\frac{1}{4}\pi i} = \sqrt{2}e \times e^{-\frac{7}{12}\pi i}. \quad (3.4)$$

$$|z_1 \times z_2| = \sqrt{2}e, \operatorname{Arg}(z_1 \times z_2) = +\frac{17}{12}\pi.$$

5.

$$z_1 + z_2 = \frac{e}{2} - \frac{\sqrt{3}e}{2}i + (1 - i) \quad (3.5)$$

$$\text{よ} \text{り}, \operatorname{Re}(z_1 + z_2) = \frac{1}{2}e + 1, \operatorname{Im}(z_1 + z_2) = -\frac{\sqrt{3}}{2}e - 1.$$

3.2 実部, 虚部, 絶対値, 偏角

略解

1. $|z| = \sqrt{1^2 + (\sqrt{3})^2} = 2$, $\tan(\operatorname{Arg} z) = \frac{\sqrt{3}}{1}$ より $\operatorname{Arg} z = \frac{\pi}{3}$.

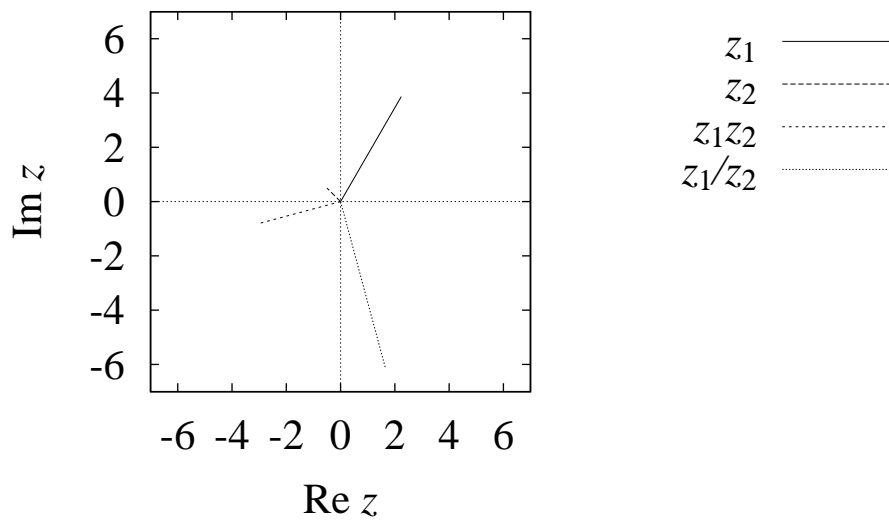
2. $z = e^2 e^{\frac{\pi}{6}i} = e^2 (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) = \frac{\sqrt{3}e^2}{2} + i \cdot \frac{e^2}{2}$.

3. $z = 2 + 0 \cdot i = 2e^{0i}$.

4. $z = 0 - i = 1 \cdot e^{\frac{3}{2}\pi i}$.

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略解

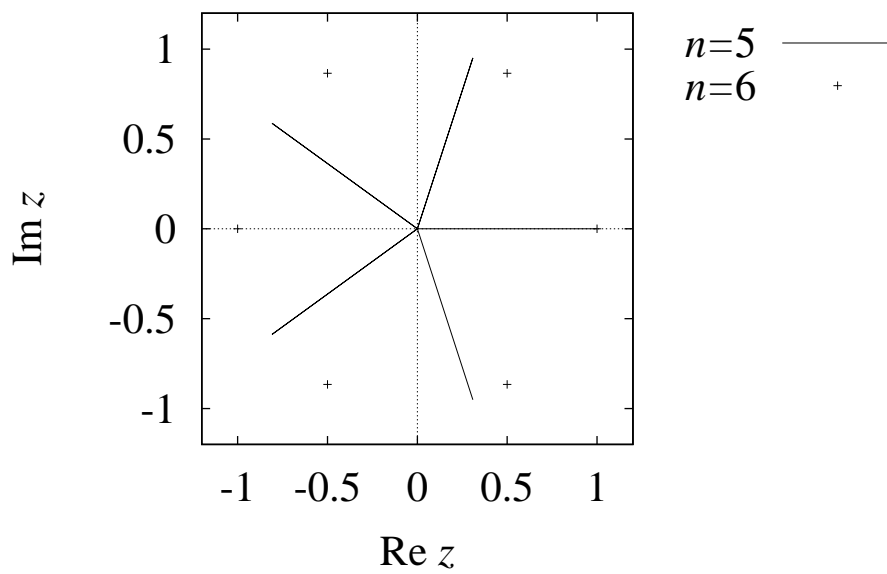


- 1.
2. $z_1 = 2\sqrt{5}e^{\frac{1}{3}\pi i}$, $z_2 = \frac{1}{\sqrt{2}}e^{\frac{3}{4}\pi i}$.
3. $z_1 z_2 = \sqrt{10}e^{\frac{13}{12}\pi i}$.
4. $z_1/z_2 = 2\sqrt{10}e^{\frac{19}{12}\pi i}$.

3.4 ドモアブルの公式 (やや難しいかも)

略解

1. 正しい式 $e^{in\theta} = (e^{i\theta})^n$ の両辺を, オイラーの公式を用いて \sin, \cos で表せばよい.
2. 3倍角の公式は $\cos 3\theta = -3\cos\theta\sin^2\theta + \cos^3\theta$, $\sin 3\theta = -\sin^3\theta + 3\cos^2\theta\sin\theta$. 他の形もあります.
3. $\left(e^{\frac{2\pi m}{n}i}\right)^n = (e^{2\pi i})^m = 1^m = 1$.



4.

5. $z = e^{\frac{2\pi}{n}i}$ とおく. 等比級数の公式より, 問題の和は,

$$\sum_{m=0}^{n-1} z^m = \frac{1 - z^n}{1 - z} = 0. \tag{3.6}$$