

微積分 演習 (略解) (情報メディア学科 1 年次科目)

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12 多変数の積分

12.1 お奨め問題

略解

1. 3.

$$2. \int_0^1 x \, dx = \frac{1}{2}.$$

$$3. \int_0^1 \left\{ \int_0^x (x^2 + y^2) dy \right\} dx = \frac{1}{3}.$$

12.2 積分順序の交換

略解

$$1. \int_0^1 \left\{ \int_{y^2}^y f(x, y) dx \right\} dy.$$

2.

$$\int_0^{1/2} \left\{ \int_x^{1-x} f(x, y) dy \right\} dx = \int_0^{1/2} \left\{ \int_0^y f(x, y) dx \right\} dy + \int_{1/2}^1 \left\{ \int_0^{1-y} f(x, y) dx \right\} dy$$

12.3 累次積分による重積分

略解

$$1. \int_0^1 \left\{ \int_{x^3}^{x^2} x \, dy \right\} dx = \int_0^1 (x^3 - x^4) dx = \frac{1}{20}.$$

$$2. \int_0^1 \left\{ \int_0^{\sqrt{1-x^2}} (2x + 3y) \, dy \right\} dx = \int_0^1 \left[-\frac{2}{3}(1-x^2)^{3/2} + \frac{3}{2}x - \frac{1}{2}x^3 \right] dx = \frac{5}{3}.$$

$$3. \int_0^1 \left\{ \int_0^y \sqrt{4y^2 - x^2} \, dx \right\} dy = 4 \int_0^1 y^2 \, dy \times \int_0^{\pi/6} \cos^2 t \, dt = \frac{\pi}{9} + \frac{\sqrt{3}}{6}. \text{ (やってみると, } x, y \text{ の累次積分の順序はこのほうが楽)}$$

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略解

1. $y = x^2$ と $y = 1$ の交点は $(\pm 1, 1)$ なので, $\int_0^1 \left\{ \int_{x^2}^1 1 \, dy \right\} dx = \frac{2}{3}$.

2. $\int_0^1 1 \, dx - \int_0^1 x^2 \, dx = \frac{2}{3}$.