チーム [ ] 学籍番号 [ ] 氏名 [ ] ______/12

確率統計☆演習 ITrial L09

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1

確率変数 X は二項分布 $\mathrm{B}(6, 0.2)$ にしたがう.

1. 確率 $P(X = 4)$ を求めよう.
2. 母平均値 $\mathrm{E}[X]$ を求めよう.
3. 母分散 $\mathrm{V}[X]$ を求めよう.

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確率変数 $X_1, \dots, X_{100}$ は $\mu = \mathrm{E}[X_i] = 3, \sigma^2 = \mathrm{V}[X_i] = 7$ の独立同分布に従う.
次の確率変数の母平均値, 母分散を答えよう.

1. 確率変数 $A = \frac{1}{100}(X_1 + X_2 + X_3 + \cdots + X_{100})$
2. 確率変数 $B = \frac{1}{10}(X_1 + X_2 + X_3 + \cdots + X_{100} - 100 \cdot 3)$

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確率変数 X, Y は独立で, $\mathrm{E}[X] = 2, \mathrm{V}[X] = 3, \mathrm{E}[Y] = 10, \mathrm{V}[Y] = 30$ を満たす.

1. $\mathrm{E}[4X + 7Y + 3]$ を求めよう.
2. $\mathrm{V}[2X - Y]$ を求めよう.
3. $\mathrm{E}[X^2Y]$ を求めよう.

12 点満点. × N:NG ワード/アイデア, × P:過程なし, ×か:考え方の誤り, ×き:記号の誤り, ×け:計算ミス

略解

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1. ${}_6\mathrm{C}_4 \cdot 0.2^4(1-0.2)^2 = 0.01536 = \frac{15\cdot 16}{5^6} = \frac{48}{5^5}$
2. $6 \times 0.2 = 1.2.$
3. $6 \times 0.2 \times (1-0.2) = 0.96$

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母平均値は, $\mathrm{E}[aX_i + bX_j + c] = a\mathrm{E}[X_i] + b\mathrm{E}[Y_j] + c$, 母分散は, $X_i, X_j (i \neq j)$ は独立なので $\mathrm{V}[aX_i + bX_j + c] = a^2\mathrm{V}[X_i] + b^2\mathrm{V}[X_j]$ の式を繰り返し使えば求められる.

1. $\mathrm{E}[A] = \frac{1}{100}(\mathrm{E}[X_1] + \cdots + \mathrm{E}[X_{100}]) = \frac{1}{100}(\mu + \cdots + \mu) = \mu = 3.$ $\mathrm{V}[A] = (\frac{1}{100})^2(\mathrm{V}[X_1] + \cdots + \mathrm{V}[X_{100}]) = \frac{1}{100^2}(\sigma^2 + \cdots + \sigma^2) = \frac{\sigma^2}{100}.$
2. $\mathrm{E}[B] = \frac{1}{10}(\mathrm{E}[X_1] + \cdots + \mathrm{E}[X_{100}]) - \frac{100\cdot\mu}{10}\mathrm{E}[1] = \frac{100\mu}{10} - \frac{100\mu}{10} = 0.$
   $\mathrm{V}[B] = (\frac{1}{10})^2(\mathrm{V}[X_1] + \cdots + \mathrm{V}[X_{100}]) = \frac{100\sigma^2}{100} = \sigma^2 = 7.$

なお, $W = \frac{\frac{1}{n}(X_1 + \cdots + X_n) - \mu}{\sqrt{\sigma^2/n}}$ は, $\mathrm{E}[W] = 0, \mathrm{V}[W] = 1$ にしたがう.

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1. $\mathrm{E}[4X + 7Y + 3] = 4\mathrm{E}[X] + 7\mathrm{E}[Y] + 3 = 81.$
2. $\mathrm{V}[2X - Y] = 2^2\mathrm{V}[X] + (-1)^2\mathrm{V}[Y] = 42.$
3. $\mathrm{E}[X^2Y] = \mathrm{E}[X^2] \times \mathrm{E}[Y] = (\mathrm{V}[X] + \mathrm{E}[X]^2) \times \mathrm{E}[Y] = 70.$