4月7日まとめ　　t９９００２２　大坪　哲雄

・四章　角運動量保存の法則

・中心力

$P = m\dot{r}$

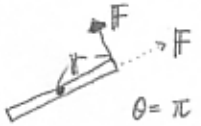
$F = m\ddot{r} = \dot{P}$

両辺に×r

$r \times F = r \times \dot{P}$　　(1)

(1)の右辺 $= \frac{d}{dt}(r \times P) - \dot{r} \times P$

$N = \dot{L}$　　L；角運動量　　(5)



$\theta = \pi$

$r \times F = 0$

$N = 0$ (例えば $F \parallel r$ のとき)

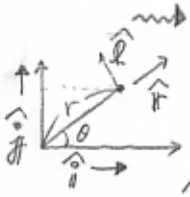
$\rightsquigarrow \dot{L} = 0$

$\rightsquigarrow$ Lは一定

$L = r \times P = m\, r \times \dot{r}$

$\rightsquigarrow r \cdot L = 0$　　Lに直交する平面内

$\dot{r} \cdot L = 0$　　速度に〃



$r = \hat{i}\, r\cos\theta + j\, r\sin\theta$

$\dot{r} = \hat{i}\,(\dot{r}\cos\theta - r\sin\theta\,\dot{\theta}) + \hat{j}\,(\cdots\cdots)$

$\ddot{r} = \hat{i}\,(\ddot{r}\cos\theta - 2\dot{r}\sin\theta\,\dot{\theta} - r\sin\theta\,\ddot{\theta}) + j\,(\cdots\cdots)$

$= (\ddot{r} - r\dot{\theta}^2)\underbrace{(\hat{i}\cdot\cos\theta - \hat{j}\cdot\sin\theta)}_{\hat{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underbrace{(-\hat{i}\sin\theta + \hat{j}\cdot\cos\theta)}_{\hat{l}}$

・運動方程式

$m\ddot{r} = F_r\,\hat{r} + F_\theta\,\hat{l}$

$\begin{cases} m(\ddot{r} - r\dot{\theta}^2) = F r \\ m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \end{cases}$

$\dot{L} = mr \times \dot{r}$

$|L| = m r \cdot r\dot{\theta} = m r^2 \dot{\theta}$

θ を消去して

$m\ddot{r} - m r \left(\frac{|L|}{m r^2}\right)^2 = F r$

$|L|$ = 一定 $\Leftrightarrow$ 中心力 $\Leftrightarrow F_\theta = 0$

$\frac{d}{dt}\left(m r^2 \dot{\theta}\right) = 0 = m\left(2r\dot{r}\dot{\theta} + r^2\ddot{\theta}\right) = 0$