T990024　　大本　悠介

$$\theta = \pm |L| \int^{r} \frac{dr}{r\sqrt{2mEr^2 + 2m\alpha r - |\vec{L}|^2}}$$

r = $\frac{1}{u}$ とおくと　dr = $-\frac{du}{u^2}$

$$\theta = \pm \int^{r} \frac{du}{\frac{2mE|\vec{L}|^2 + m^2\alpha^2}{|\vec{L}|^4} - (u - \frac{m\alpha}{|\vec{L}|^2})^2}$$

$\int \frac{dx}{a^2 - x^2} = \arcsin\frac{x}{a} + C$ より

$$\theta = \pm \arcsin \frac{|\vec{L}|^2 - m\alpha r}{rm\alpha\sqrt{1 + \frac{2E|\vec{L}|^2}{m\alpha^2}}} + C$$

$$\varepsilon \equiv \sqrt{1 + \frac{2E|\vec{L}|^2}{m\alpha^2}}$$　　より

$$\theta - C = \pm \arcsin(\frac{m\alpha r - |\vec{L}|^2}{\varepsilon m\alpha r})$$

sin(arcsinx) = x より

$$\sin(\theta - C) = \pm(\frac{m\alpha r - |\vec{L}|^2}{\varepsilon m\alpha r})$$

$$r = \frac{\frac{|\vec{L}|^2}{m\alpha}}{1 \pm \varepsilon \sin(\theta - C)}$$

$$\theta_0 - C = \pm\frac{\pi}{2}$$　より

$$r(\theta) = \frac{\lambda(1+\varepsilon)}{1 + \varepsilon\cos(\theta - \theta_0)}$$

$$\lambda \equiv \frac{|\vec{L}|^2}{m\alpha} \cdot \frac{1}{1+\varepsilon}$$

$\theta_0 = 0$, $\varepsilon = 1$ のとき

r = $\dfrac{2\lambda}{1+\cos\theta}$

x = r$\cos\theta$, y = r$\sin\theta$ より

$r^2 = x^2 + y^2 = 4\lambda^2 - 4\lambda x + x^2$

よって $y^2 + 4\lambda x = 4\lambda^2$

起動の長半径 a および短半径 b をλ, εで表すと

$a = \dfrac{\lambda}{1-\varepsilon}, b = \lambda\sqrt{\dfrac{1+\varepsilon}{1-\varepsilon}}$

$\dfrac{b}{a} = \sqrt{1-\varepsilon^2}$

$a = \dfrac{\lambda}{1-\varepsilon}$

$= -\dfrac{\alpha}{2E} = \dfrac{\alpha}{2|E|}$

$b = \lambda\sqrt{\dfrac{1+\varepsilon}{1-\varepsilon}}$

$= \dfrac{|\vec{L}|}{\sqrt{2m|E|}}$

$E = -\dfrac{\alpha}{2a}$

$E = T + V, V(r) = -\dfrac{\alpha}{r}$

$V(a) = -\dfrac{\alpha}{a}$ より

$E = \dfrac{1}{2}V(a) = -T$