

## 微積分 演習 (略解) (情報メディア学科1年次科目)

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### 7 多変数関数の微分

#### 7.1 お奨め問題セレクション

略解

1.  $\frac{\partial f}{\partial x}(x, y) = 2x, \frac{\partial f}{\partial x}(-1, 1) = -2, \frac{\partial f}{\partial y}(x, y) = 1, \frac{\partial f}{\partial y}(-1, 1) = 1.$
2.  $z - 2 = -2(x + 1) + (y - 1).$
3.  $f_x(x, y) = 5x^4 + 12x^3y^2, f_{xx}(x, y) = 20x^3 + 36x^2y^2, f_{xy}(x, y) = 24x^3y, f_y(x, y) = 6x^4y + 4y^3, f_{yx}(x, y) = 24x^3y, f_{yy}(x, y) = 6x^4 + 12y^2.$

#### 7.2 偏導関数

略解

1.  $f_x(-1, 1) = -\frac{1}{\sqrt{2}}, f_y(-1, 1) = \frac{1}{\sqrt{2}}, z - \sqrt{2} = \frac{1}{\sqrt{2}}(-(x + 1) + 1(y - 1)).$
2.  $f_x(-1, 1) = -2\pi, f_y(-1, 1) = 3\pi, z - 0 = -2\pi(x + 1) + 3\pi(y - 1).$
3.  $f_x(-1, 1) = -i\pi e^{-\frac{1}{2}}, f_y(-1, 1) = (1 + i\pi)e^{-\frac{1}{2}}.$

#### 7.3 高階偏微分

略解

1.  $f_x(x, y) = 8x + 2y - 6, f_y(x, y) = 2x + 2y + 6, f_{xx}(x, y) = 8, f_{xy}(x, y) = f_{yx}(x, y) = 2, f_{yy}(x, y) = 2.$
2.  $f_x(x, y) = 2xy^2 + \frac{y}{(2+x)^2}, f_y(x, y) = 2x^2y - \frac{1}{(2+x)}, f_{xx}(x, y) = 2y^2 + \frac{-2y}{(2+x)^3}, f_{xy}(x, y) = 4xy + \frac{1}{(2+x)^2}, f_{yy}(x, y) = 2x^2.$

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## 7.4 テイラー展開の復習

略解  $f(1) = 1, f'(1) = -3, f''(1) = +9$  より,

1.  $y - 1 = -3(x - 1).$

2.  $y - 1 = \frac{9}{2}(x - 1)^2 - 3(x - 1).$

3.

$$f(x) = e^{-3(x-3)} = \sum_{k=0}^{\infty} \frac{1}{k!} (-3(x-1))^k = \sum_{k=0}^{\infty} \frac{(-3)^k}{k!} (x-1)^k. \quad (7.1)$$

4.  $f(1.1) \simeq 1 + (-3) \cdot 0.1 + \frac{(-3)^2}{2} (0.1)^2 = 1 - 0.3 + 0.045 = 0.745.$